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THE MONETARY MULTIPLIER AND THE MONETARY MODEL.

by

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Preface.

Recently Dr. Holtrop published a paper in which he clearly explains the monetary model of the Nederlandsche Bank of which he was the president for over two decades. Further he examines the developments which occured in the Dutch economy during the period 1954-1969 and the effects of the actions taken by the Nederlandsche Bank with respect to its monetary policy.[1)]

We assume that after the frequent discussions, in which Holtrop also participated, concerning the similarities but especially with respect to the controversies which could exist between "the real multiplier" and "the monetary multiplier", many students have the unsatisfactory feeling that they are unable to penetrate as yet into the core of the problem.

By "the real multiplier" is meant here the income-multiplier for a change in the autonomous expenditures and "the monetary multiplier" is the income-multiplier for a change in the stock of money. By means of this study we will try to throw some light on the asymmetrical approaches of the real and monetary multipliers.

In order to bridge the assumed controversies and to bring out unsuspected contrasts it appears desirably to make a correct analysis of both multiplier theories.

In addition, the presentation of the model called our attention again to the problem of the lack of dimensional consistency. This problem has already been delt with in the past

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1) M.W. Holtrop: On the effectiveness of Monetary policy: The experience of the Netherlands in the years 1954-69. Journal of Money, Credit, and Banking, vol. IV, May, 1972, pp. 283-311.
Also published in: Money in an open economy, Leiden, 1972, pp. 221-254.

and other writers have offered their solutions too.[2)]

This study intends to further examine the policy consequences that in the opinion of Holtrop can be determined as a result of his system.

As a matter of fact the statistical verification, such as presented by Holtrop, is meeting a needed opposition. The purpose of this article is to give a critical interpretation of the quantitative approach, by means of the monetary model, of the monetary policy as pursued by the Nederlandsche Bank. Chapter I provides a critical review of the model underlying Holtrops monetary analysis.
Chapter II consists of an analysis of Holtrop's statistical verification.

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2) H.W.J. Bosman: Een standaardwerk over de monetaire problematiek. Maandschrift Economie, February, 1961, pp. 258-260.
F.J. de Jong: Dimensieanalyse in de economie. De Economist, 110, nr. 1/2, 1962, pp. 1-200.
G.A. Kessler: Monetaire model en dimensieanalyse. De Economist, 111, nr. 7/8, 1963, pp. 481-496.
F.J. de Jong: De monetaire analyse van de Nederlandsche Bank. Tijdschrift voor documentatie en voorlichting, 1965, 4, pp. 424-448 and 5, pp. 569-585.
F.J. de Jong: Dimensional analysis for economists. Amsterdam, 1967, pp. 72.
Apart from the above-mentioned publications I wrote a non published critic on the generally known model of J.J. Polak and W.H. White: The effect of income expansion on the quantity of money. Staff Papers, I.M.F., vol. IV, 1955, pp. 398-433.
This model is perhaps a still more instructive example of the fact that dimensional inconsistency can involve fully unlogic and unacceptable conclusions.
Also H.G. Johnson's recent publication about this subject: Inflation and the monetarist controversy. Amsterdam, 1972, pp. 100-105, suffers from inconsistencies.

I. A COMPARITIVE ANALYSIS.

§1. The real multiplier.

The central point in the Keynesian line of thinking is the analysis of the real multiplier.
Hereby a relationship is constituted between an initial change in the level of one or more of the autonomous demand components and, as a result of it, an arising change in the level of national income.
This formulation underlies the so-called "equality approach" into which we shall discuss later on.
The national income that is equal to the value of the national product will change and in turn will create a change in the level of consumptive expenditures.
With regard to an explanation of the level of investment, two theories of investment demand can be distinguished both of them entirely fitting into the Keynesian demand analysis. The first one based on the theory of the "marginal efficiency of capital" points out that a negative connection exists between the level of investment and the level of the rate of interest on the capital market.[3)]
Introducing this relation causes the necessity to explain the level of this rate of interest. When we do so, by means of the liquidity preference theory of Keynes, we have a complete model with a real and a monetary sector linked up with each other by the rate of interest. Graphically this model can be presented with the aid of the well known IS- and LM-curves.
The real multiplier now becomes smaller because of the influence of three important factors: the interest-elasticity

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3) Putting it more precise, there exists a negative relationship between the required stock of capital goods and the rate of interest.
See among others: G. Ackley: Macro economic Theory, New York, 1961, chapter XVII.

of the expenditures and of the inactive cash-balances, both of them being negative, and the positive cash-quota. It would appear from this that a not entirely flexible money-supply has a stabilizing influence on changes in the level of national income.

Another explanation for the level of investment can be found in the acceleration principle. Hereby we assume that a change in the level of the demand for consumer goods will, in adjusting the production capacity, lead to a change in the level of investment demand. This can be expressed by considering the level of investment to be a positive function of the level of income. In the real multiplier the investment-elasticity for a change in the level of income will now also play a part along with the marginal propensity to save so that the value of this multiplier will increase.

Consequently, the accelerator has a destabilizing effect on any probable changes in the level of national income. However, in this case it has to be considered that the monetary mechanism supplies the required financial funds in a flexible way.

From the above description it appears that both theories of investment affect the value of the real multiplier. The above mentioned refinements, with respect to investment-behaviour. stay out of consideration. Because they do not violate the analysis of our subject and in order to be able to give an easily understandable description of our subject, we assume in the forthcoming argumentation that the level of investment will not be determined by endogeneous variables but only by exogenous factors.

Now we return to the description of the working of our simple model. Assuming that through the existence of savings every additional change in the level of consumption is smaller than the change in the level of income of which it was the result this multiplier-process of income and consumption will be continued although gradually decreasing in intensity. Thus the change in the level of income will finally attain the following value:

$$\Delta Y = \frac{1}{s} \Delta \bar{U} \qquad (1.1.1)$$

in which

ΔY represents the change in the level of national income,

and

$\Delta \bar{U}$ the initial change in the autonomous expenditures,

and finally

s the change in the level of savings as a rate of the change in the level of national income.

If we adopt the assumption that a part of the expenditures addresses itself to goods produced in foreign countries then a change in the level op expenditures,as far as this part is concerned,will not result in a change in the level of national income but in a change in the level of imports.
The ultimate change in the level of income will now become smaller, namely

$$\Delta Y = \frac{1}{s+m} \Delta \bar{U} \qquad (1.1.2)$$

in which

m represents the change in the level of imports as a rate of the change in the level of national income.

An implicit assumption underlying the above multiplier analysis is that all changes in the level of income are financed adequately in a completely elastic way by changes in the same direction in the supply of money.

Considering both ratios s and m a bit closer it is evident that both of them show a relationship between changes into two flows so that the value of the ratios is independent of

the length of the period under consideration. In order to get a better insight in this matter, let us denote the dimensions of both ratios:

$$s = \frac{\Delta S}{\Delta Y} \in \frac{[MT^{-1}]}{[MT^{-1}]} = [1] \quad 4)$$

and

$$m = \frac{\Delta Im}{\Delta Y} \in \frac{[MT^{-1}]}{[MT^{-1}]} = [1]$$

in which ΔIm represents the change in the level of imports. It is quite clear that both ratios are to be considered as dimensionless entities.

§2. The monetary multiplier.

Let us now pay attention to the analysis of the monetary multiplier playing an important part in the theories in which the level of income, contrary to the assumptions of the Keynesian analysis mentioned above, is determined ultimately by the existing stock of money.
In the analysis of the monetary multiplier a relationship is stated between an initial change in the supply of money and a consequently occuring change in the level of production and income. Making use of this definition the value of the monetary multiplier will thus be the factor which multiplied by the initial change in the supply of money will yield the ultimate change in the level of income. The definition, used above, which starts again from the so-called "equality approach", is emphasized because this description is deviating from the usual formulation of the monetary multiplier. Later on we shall come back to the consequences of this subject. The process, according to which an initial change in the supply of money brings about the ultimate change in the level of income,

4) See for an extensive explanation of the dimensions in macro-economic theory: F.J. de Jong: Dimensional analysis for economists, Amsterdam, 1967.

can be analyzed starting from the following two descriptions.

This analysis is based on the fact that an initial change in the supply of money involves an equal change in income [5]. The period over which this change in income is measured is called the income-period. If the change in the demand for money is a fixed rate "k" of the change in the level of income, then a multiplier-process is generated as a result of an initial change in the supply of money. The result of this is that the change in the level of income in a period relevant for the analysis is becoming equal to the initial change in the supply of money multiplied by the reciprocal of the mariginal cash-quata k.

At the same time following the first income period, the multiplier-process described in the preceding paragraph has been started on the basis of the marginal propensity to save in consequence of the initiated change in the level of expenditures. For this reason the change in the level of income in the period under consideration in the analysis will finally become equal to the initial change in the supply of money multiplied by the reciprocal-values of the marginal propensity to save s and the marginal cash-quota k. Adding to the above it should be noted again that the multiplier-process based on the marginal propensity to save will only develop undisturbed if the supply of money adjusts in a fully elastic way to the changing demand for money resulting from the appearing changes in the level of income.[6]

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5) It is true that under circumstances such as described by Keynes, it is possible that changes in the quantity of money will only involve contrary changes in the velocity of circulation of money. It is quite natural that such an extremely deflationary situation, though being no more actual for the last decades, in which a monetary policy must be fully inefficient indeed, irrespective of the effects such as those resulting from the real cash-balances and wealth redistribution, cannot be a starting point for an analysis of the monetary multiplier.

6) We find a similar description of the multiplier-process by G.A. Kessler: Monetair evenwicht en betalingsbalans-evenwicht. Leiden, 1958, p. 167.

The outcome of the above process for the change in the level of income is:

$$\Delta Y = \frac{1}{s.k} \Delta \bar{M} \qquad (1.2.1)$$

in which

k represents the change in the supply of money as a rate of the change in the level of income.

and

$\Delta \bar{M}$ the initial change in the supply of money.

The above-mentioned description of the multiplier-process follows from the cash-balances approach of the quantity-theory that starts from a cash-quota required by individuals.[7] If the actual cash-quota would deviate from the desired one through an initial change in the stock of money, or in other words, if the marginal cash-quota deviates from the structural one then individuals will try to restore the desired cash-quota by adapting the level of their expenditures. Formulated this way, the process on the basis of the cash-quota can indeed be considered as an equilibrium restoring multiplier-process.

Let us now assume,as the alternative of the above line of thinking, that,the proper multiplier-process is only developing on the basis of the marginal propensity to save and that every change in the level of expenditures will adequately be financed by an equal change in the supply of money. An initial change in the supply of money will have led to an equal change in the level of income measured over one income-period.

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7) See for this among others:
A. Marshall: Money, Credit and Commerce. London, 1923.
A.C. Pigou: The value of money. Quarterly Journal of Economics, vol. XXXII, 1917.
M. Friedman: A theoretical framework for monetary analysis. Journal of political economy, 1970, vol. 78, pp. 200.

As a consequence of this change in the level of expenditures a multiplier-process will now be generated on the basis of the marginal propensity to save if the supply of money adjusts fully elastically to the changing level of the demand for money during this process.
When this multiplier-process has come to an end, then the change in the level of income measured over one income-period, and consequently the total change in the money-supply, will be equal to the initial change in the money-supply multiplied by the reciprocal of the marginal propensity to save. The change in the stock of money is by definition equal to the change in the level of national income measured over the income-period. In any other period the change in the level of national income will be equal to the change in the stock of money multiplied by the income-velocity of circulation of money during that period. The income velocity of circulation is the reciprocal of the cash-quota.
The result of the process described in this way can be symbolized again with the aid of the equation (1.2.1). This description of the multiplier-process emphasizes the cash-quota as a reciprocal of the income-velocity of circulation of money.
In order to compare the change in the stock of money, being a stock,with the change in the level of income, which is a flow, raised by that change in the stock of money, the lenght of the period over which the change in income is measured should be defined by means of the number of income-periods which comprises this period under consideration.
If the total change in the stock of money is multiplied by the number of income-periods which comprises the period under review, i.e. the income-velocity of circulation 1/k in that period, we will get by definition the change in the level of income measured over the period in question. In this description the factor k is only attributed an ex-post character so that there is no question of a difference between the structural and marginal cash-quota in this analysis.
Therefore the reciprocal of the cash-quota can only be mentioned as a multiplier in the sense of a relationship between impulse

and effect.
This merely comparative static interpretation is following from the so-called income-version of the equation of exchange of which particularly Wicksell, Schumpeter and the young Keynes were the interpreters.[8)]
Before entering into the implications of a choice between both versions of the quantity-theory some conclusions are drawn with regard to the similarities and differences between the real multiplier and the monetary multiplier.
For that purpose we revert to the deviation of our formulation, mentioned earlier, from the one that is used mostly, among others by Holtrop and Kessler.[9)]
From the above it clearly appears that the total change in the supply of money was equal to the initial change in the supply of money multiplied by the reciprocal of the marginal propensity to save:

$$\Delta M = \frac{1}{s} \Delta \bar{M} \qquad (1.2.2)$$

in which

ΔM represents the total change in the domestic stock of money.

Among others, Kessler is defining the monetary multiplier as the factor with which the total change in the supply of money, the "monetary impulse", has to be multiplied in order to get the change in the level of money-income with which he starts from the so-callled "identity approach".
From the equations (1.2.1) and (1.2.2) it will immediatly become evident that the monetary multiplier will get the value 1/k following the "indentity approach".

8) K. Wicksell: Interest and Prices. R.F. Kahn, trans., London, 1936.
J. Schumpeter: Das Sozialprodukt und die Rechenpfennige. Archiv für Sozialwissenschaft und Sozialpolitik, vol. 44, 1917/18.
J.M. Keynes: A Treatise on Money. London, 1930.

9) M.W. Holtrop: Money in an open economy. pp. 162-163 and p.225.
G.A. Kessler: op. cit. p. 164.

In this way we are creating a non-existing asymmetry between the conceptions of the real and of the monetary multiplier. In case we reject the "equality approach", whereby the change in the level of an endogeneous variable is measured in terms of an initial change in the level of one of the autonomous factors, but we do use the "identity approach" for the real multiplier in order to maintain the symmetry, then the real multiplier is becoming equal to 1. Because the change ultimately realized in the level of income will by definition be euqal to the total change in the level of expenditures. Then the two multipliers can be read from the following identities:

$$\Delta Y = \Delta U \qquad (1.2.3)$$

and

$$\Delta Y = \frac{1}{k} \Delta M \qquad (1.2.4)$$

in which

ΔU represents the total change in the level of the expenditures.

The real multiplier is now:

$$\frac{\Delta Y}{\Delta U} = 1$$

while the monetary multiplier now is as follows:

$$\frac{Y}{M} = \frac{1}{k} .$$

It must be noted that the multiplier defined in this way cannot be considered as more than a tautological notation of the iden-

tities (1.2.3) and (1.2.4).[10)]
This does not at all give an insight into the process that we have described.
If we start from the "equality approach" in analyzing the real and monetary multiplier, then the values of both multiplier wil change from

$1 \rightarrow \frac{1}{s}$ for the real multiplier and from

$\frac{1}{k} \rightarrow \frac{1}{s.k}$ for the monetary multiplier.

During the multiplier-process,on the basis of the marginal propensity to save,the supply of money has to be adjusted in both cases to the level of the money demand from $\Delta\bar{M} \rightarrow \Delta M$ in the same entirely elastic way.
After having stated explicitly in this way the assumptions with respect to the "equality approach" respectively to the "identity approach" we should go further into the question of the factor of time.Analyzing the multiplier,on the basis of the marginal propensity to save,we assume implicitly to base it on the income-period. The cash-quota counting for this period is by definition equal to 1.
If we do not start explicitly from this period,or the analysis is even based on another period, then the income-velocity of circulation and consequently the cash-quota is still not playing a part in the real multiplier because in the analysis of the spending process impulse and effect are measured over the same period.
If we measure the ratio between the autonomous impulse of expenditures and the change in the level of income which was the result of it, then the factor of time is eliminated out of the dimension and a real multiplier will come about which has

10) See p.e.
T.F. Dernburg and D.M. McDougall: Macro economics. New York, 1972, p. 173.
J. Aschheim and Ching-Yao Hsieh: Macro economics: income and monetary theory. Columbus, 1969, p. 166.

to be considered as a dimensionless entity.
In case the impulse is an entity independent of the factor of time, but the factor of time does play a part in the dimension of the effect generated by this impulse, it will be clear then that the factor of time can be found in the dimension of the multiplier as a ratio between impulse and effect in order to make both of them comparable for a certain period.
This is the case in the multiplier-analysis based on the marginal cash-quota k. From the interpretation of the income-approach of the quantity-theory it appears clearly that this factor k only establishes the value of the change in the level of income as a consequence of a change in the stock of money for the period which is under consideration. The value of the reciprocal of the factor k indicates the number of income-periods being comprised by the period in question. Should the analysis of the monetary multiplier be carried out in periods to the extent of the income-period then the factor k should get the value 1 and the monetary multiplier will change into the real multiplier.
Summarizing it can be stated that the real multiplier and the monetary multiplier are describing the same process and that their values deviate from each other as a result of the assumption underlying both analyses. The first difference in assumption is, such as we have demonstrated, the fact that respectively the "equality approach" is underlying the real multiplier and the "identity approach" is underlying the monetary multiplier.
The second difference is that the factor k in the analysis of the monetary multiplier only determines over which period the change in income has to be measured because the impulse has to be considered as a stock and the effect as a flow. In the analysis of the real multiplier in which impulse and effect are of the same dimension and both of them must be considered as a flow, the lenght of the period under consideration does not influence the level of the multiplier which as a ratio between effect and impulse turns out to be a dimensionless entity.

Without entering further into this matter in the frame-work of this article, we finally make some remarks about the implications of a choice between both approaches of the quantity-theory. From the income-approach we have learned to know the explicit treatment of the cash-quota as a factor of time. This gives the advantage that hereafter, in a simple way, attention can be paid to the cause of the problem of the dimensional inconsistency of the monetary model of the Nederlandsche Bank. Moverover, the income-approach, being a comparative static analysis, fits in better with the equally comparative static Keynesian analysis of effective demand. As we demonstrated above the possibility of a consequent comparative multiplier analysis will arise.
An important and in our opinion also decisive advantage of the cash-balances approach is the fact that it fits in better with Keynes' liquidity preference theory and with the modern developments in the monetary theory such as the theory of the optimum cash-balances [11], the portfolio-balance theory, the theory of the monetary influence of non-monetary financial intermediaries [12], and with the restatement of the quantity theory [13].

Although we have demonstrated the similarities and the differences which exist between the real multiplier and the monetary multiplier by emphasizing both assumptions underlying the multiplier analysis, there still remains the question whether we must prefer from a theoretical point of view the "equality approach" or the "identity approach".
It may be stated that apart from the period under consideration, a multiplier-analysis will only be significant if it indicates

11) W.J. Baumol: The transactions demand for cash: an inventory theoretical approach. Quarterly Journal of Economics, vo. 66, November, 1952.

12) J. Tobin: Money, capital, and other stores of value. American Economic Review, 51, May, 1961.
J.G. Gurley and E.S. Shaw: Money in a theory of finance. Washington, 1960.

13) M. Friedman: The quantity theory of money- A restatement. Studies in the quantity theory of money, Chicago, 1956.

the relationship between a change in the level of an exogenous factor, using it as an instrument, and the change generated by this in the level of an endogeneous variable provided that there is a fully elastic supply of those variables which could otherwise frustrate this multiplier-process.
If this last condition is not fullfilled, the factors affecting the value of the multiplier have to be fitted into the multiplier itself[14].
The question that should be asked is as follows:
is it possible to consider the changes in the stock of money, from period to period, to be a direct result of a range of autonomous policy-discussions of the monetary authorities?
If this question can be answered in the affirmative, then the total monetary impulse can be involved in the analysis of the monetary multiplier and consequently the identity approach can be used.
However, if the total change in the stock of money is not the outcome of the consciously pursued policy of the monetary authorities alone, but is also brought about by effects controlled not at all or in a less effective way by the monetary authorities, it is not possible to use a monetary analysis

14) In connection herewith we think of the curbing working of a rise of the rate of interest on expenditures in the absence of an elastic supply of money and of the stimulating working of the accelerator such as was mentioned in the above reflection concerning the investment function. The real multipliers are becoming then respectively:

$\frac{1}{s+\frac{ak}{b}}$ in case of a not fully elastic supply of money and

$\frac{1}{s-d}$ in case of the working of the accelerator

in which a represents the interest-elasticity of investment,
in which b represents the interest-elasticity of inactive balances,
and d represents the income-elasticity of investments.

fruitfully based on the "identity approach" as a theoretical foundation of monetary policy. Starting from the "identity approach", the only significance that remains is to get some insight afterwards into the stability of the marginal cash-quota by using the identity (1.2.4). However, this cannot be called a multiplier analysis. Now let us try to give an answer to the above decisive question.

If the monetary authorities refuse, after having consciously induced an initial change in the stock of money, to bring about further changes in the money supply to finance elasticly the multiplier-process based on the marginal propensity to save, these changes will still be enforced. This can be effected through a transformation between certain types of near money (short-term claims on the government, local public authorities, and money creating institutions - the so called "secondary liquidities" -) and money. A further argument is the frustration with regard to the effectiveness of monetary policy caused by the activities of a well developed system of non-monetary financial intermediaries. In order to avoid these problems the monetary authorities can base the monetary analysis on changes in the total stock of liquidity comprising money and near money instead of basing oneself on changes in the stock of money alone. In this case tranformation between money and near money will not affect the total holdings of liquidity. By using this broader concept of liqidity, for which good arguments may be brought forward for the rest, the problem that exists with respect to the control of the stock of money will only be shifted.

With that broader concept of liquidity, the stock of the circulating liquidity should be controlled better but the velocity of circulation of the liquidity that is coming under this boarder definition will be subject to larger fluctuations. As the concept of money will be described in a narrower sense, the velocity of circulation will accordingly not only increase but also become more stabilized. The solution of the problem of controlling the product of the quantity of money and of the velocity of circulation, which

should be the relevant purpose for the monetary policy, is not at all brought nearer in consequence of the choice of the definition of money. Henceforth we still prefer in conformity with the usages of the Nederlandsche Bank to speak about the broader concept namely the stock of liquidity comprising money and "secondary liquidities" instead of speaking about the stock of money.

We also are confronted with the problem of the influence on the total domestic quantity of money, respectively on the total domestic stock of liquidity, resulting from the balance of payments. Special attention is paid here to the frustrations with regard to the effectiveness of the monetary policy coming from abroad under a system of fixed exchange rates and absolute convertibility. The monetary authorities really do not have the instruments at their disposal which can sufficiently restrain the influence coming from the balance of payments surpluses or deficits on the domestic liquidity holdings.
A third important frustration of the policy of the monetary authorities comes from the activities of the government. It appears from the figures in appendix I, concerning the domestic liquidity creation in behalf of the government, that the central bank has hardly been able to make use in an effective way of the instruments being at its disposal to co-ordinate the liquidity creating effect resulting from the activities of the governments own policy.

We have to arrive at the conclusion that under the given assumptions of fixed exchange rates and absolute convertibility, changes in the stock of money and even in the stock of liquidity, cannot be considered as exogenous entities to be fixed autonomously by the monetary authorities. Instead they are, for the greater part, endogeneous variables which are induced by the real multiplier process. In order to support the above argumentation with facts we have mentioned the figures in appendix I regarding changes in the stock of liquidity during periods in which there was followed and during periods in

which a conciously conducted policy by monetary authorities of restrictive credit policy was not followed. The sum of domestic monetary impulses proved to be 3.3% in years without restrictive credit policy. When restricted policy was in effect the impulses increased to 9.7%. Especially on the pro-cyclical liquidity creation in behalf of the government and on the liquidity activation monetary authorities could apparently exercise no influence. The extent of the external impulses which assert its influence on the domestic stock of liquidity through the balance of payments is excluded from tabel I because it withdraws practically entirely from the influence of the instruments of monetary authorities.

In our opinion there does not exist a clear motive to use an asymmetrical method especially in a comparative analysis between the real and the monetary multiplier. With respect to the multiplier based on the "identity approach" we arrived at the following two conclusions:

a) described this way a real analysis is lacking. In other words, there is no formulation of the process in question. The factor that comes about in the "identity approach" is considered as the multiplier of the process.

b) This multiplier based on the "identity approach" turns out to be only a ratio between the extend on the changes in two endogeneous variables.Such a ratio is not of any value at all for policy.

For this reason the choice is turning in favor of a consequent use of the "equality approach" for the real multiplier as well as for the monetary multiplier.

In such instances the influence of monetary authorities can be seen from the extent in which they can raise the marginal propensity to save above its structural value. In case the marginal propensity to save can be increased to 1 by means of forced savings, the multipliers based on the "identity" and "equality approach" will coincide.

To start a real multiplier-process,by which the level of income is affected every period, will then be made impossible by omitting a liquidity creation or dishoarding.

Now we return to our multiplier-process and abandon the assumption again, like in the case of the analysis of the real multiplier, that the total change in the level of expenditures induced by an initial change in the domestic stock of liquidity only relates to the homemade products. The total change in the domestic stock of money will now be, as in the preceeding, the resultant of the entirely elastic adjustment to changes in the demand for liquidity as a consequence of the real multiplier process that started by an initial change in the domestic stock of liquidity.

$$\Delta M = \frac{1}{s+m} \Delta \bar{M} \qquad (1.2.6)$$

The ratio between the total change in the domestic stock of liquidity and the nominal national income is reflected in the literature, taking into account the import-leakage,as follows:

$$\frac{\Delta Y}{\Delta M} = \frac{1}{k+m} \quad 15) \qquad (1.2.6)$$

By means of this equation the value of the monetary multiplier based on the "identity approach" is demonstrated at the same time.

The final equation based on the "equality approach" for an open economy can now be described as follows:

15) See among others
M.W. Holtrop: The relative responsibilities of governments and central banks in controlling inflation. Money in an open economy, Leiden, 1972, p. 163.
M.W. Holtrop: On the effectiveness of monetary policy, the experience of the Netherlands in the years 1954-1969. Journal of Money, Credit and Banking, vol. IV, May, 1972, p. 286.

$$\Delta Y = \frac{1}{(s+m)(k+m)} \Delta \bar{M} \qquad (1.2.7)$$

We do object strenuously to equation (1.2.6) as well as to equation (1.2.7).
From the dimensional analysis of the real multiplier it became evident that the ratios s and m used there were dimensionless entities. Therefore they maintained their value irrespective of the length of the period to which they were applied.
If we do analyze the dimension of the liquidity ratio k being used in the monetary multiplier-theory,

$$k = \frac{\Delta M}{\Delta M} \in \frac{[\ ]}{[MT^{-1}]} = [T]$$

then we will see from it very clearly that this ratio is of the dimension "time". Consequently this ratio can be expressed in terms of a time-unit such as a year or an income-period.
Proceeding from this dimensional analysis, it is clear that the condition of dimensional homogenity in the equations (1.2.6) and (1.2.7), a necessary but not yet satisfactory condition for consistency, has not been fulfilled.
Consequently the above-mentioned equations cannot be right.
In order to enter more deeply into equation (1.2.6) we go back to the definition (1.2.4) for a closed economy, $\Delta Y = \frac{1}{k} \Delta M$, in which the period defining character of the factor k was emphasized. The total change in expenditures leads here completely to changes in the level of the domestic income which is equal to the total change in the domestic stock of liquidity multiplied by the velocity of circulation for the period under consideration.
The same applies to an open economy. The total change in expenditures which, as far as it concerns domestic expenditures leads to changes in the level of the domestic income and, as far as it concerns expenditures in foreign countries, leads to changes in the level of imports which again is equal to the

total change in the domestic stock of liquidity multiplied by the velocity of circulation.

Thus:

$$\Delta Y + \Delta I_m = \frac{1}{k} \Delta M$$

or rewritten:

$$\Delta Y = \frac{1}{k+km} \Delta M$$

If we call the marginal propensity to import m used in the analysis of the spending process m_b and the marginal import-leakage km from the monetary analysis: m_m, so that

$$m_m = km_b \qquad (1.2.8)$$

then the equations (1.2.6) and (1.2.7) will change as follows:

$$\frac{\Delta Y}{\Delta M} = \frac{1}{k+m_m} \qquad (1.2.6')$$

and

$$\Delta Y = \frac{1}{(s+m_b)(k+m_m)} \Delta \bar{M} \qquad (1.2.7')$$

in which m_m now has, just like k, the dimension [T].
The value of m_m will consequently be defined too by the length of the period under consideration.
The logical consistency of the model has now taking care of the dimensional homogeneity of both equations as illustrated above. The question may be put again as to whether the monetary multiplier is to be made to conform to the real multiplier, if starting from the same period, as a time-unit for both multiplier analyses. Let us for that purpose turn, for the monetary multiplier, to the income period also underlying the real multiplier in which the factor k is equal to 1. We will see then the equality of the two multipliers. Hereby it should

be considered that under the assumption of a completely elastic money-supply, such as assumed in the theory of the real multiplier, an outflow of liquidity abroad is immediatly replenished and cannot play a part on defining the level of national income. This implies that $m_m = 0$. The multiplier in (1.2.7') will pass on now to the simple real multiplier as given in (1.1.2).

Conclusion.

Summarizing,it can be stated that both the real multiplier and the monetary multiplier describe the same process.
In the first analysis attention was payed to the expenditures whereas in the last analysis the financing of these expenditures is emphasized. The discrepancies, existing between the value of the real multiplier and the value of the monetary multiplier such as described in the literature, are to be found in the assumptions underlying both analyses whether explicit or not. It is customary that the analysis of the real multiplier process is based on the "equality approach".
The analysis of the monetary multiplier is presented according to the "identity approach".
It also can be stated that in the analysis of the real multiplier the length of the unit-period under consideration does not play a part because the real multiplier as a ratio of a change in the level of income and a change in the level of expenditures is a dimensionless entity. In the monetary multiplier, indicating the ratio between a change in the level of income and a change in the quantity of money, the factor "time" does play a part because the monetary multiplier is a ratio between a flow and a stock.
In the time unit, the income-period, both of these entities will be equal to each other. In any other period both entities become comparable if the multiplier indicates how many income-periods are comprised by the period under consideration.
Regarding the "identity approach" underlying the monetary analysis, we have already brought forward some objections.

That an other period than the income-period for the monetary multiplier is used relates to the fact that only statistical data with respect to calendar periods are available.

II. THE MONETARY MODEL OF THE NEDERLANDSCHE BANK.

§1. The monetary model.

In chapter I we arrived at the conclusion that a significant monetary analysis has not only to refer to the ex-post calculation of the marginal liquidity-ratio from the identities as used by the Nederlandsche Bank but also to the total process taking place between an initial autonomous change in the supply of money and the ultimate change in the level of income. Consequently the analysis based on the "equality approach" was preferred to the system of identities resulting from using the "identity approach". In this section we shall not further reject the use of identities.
We shall critically discuss the analysis and the conclusions drawn from it, which Holtrop formulates in his publication mentioned in the beginning of this article, with reference to our analysis in chapter I with regard to the marginal propensity to import. Following through now on Holtrop's idea that the monetary impulses can be divided in domestic and external monetary impulses, whereby the domestic monetary impulse consists of the domestic creation of liquidity and of the liquidity activation, we can reformulate our identity (1.2.6') as follows:

$$\Delta Y = \frac{D+E}{k+m_m} \qquad (2.1.1)$$

in which D represents the sum of domestic monetary impulses, in this case the net creation of liquidity Lcr and the net liquidity activation Lact. E represents the sum of the external monetary impulses.

The withdrawal of liquidity out of the domestic circulation, resulting from payments for additional imports as a result of a rise in income induced by monetary impulses, can be defined as follows:

$$m_m \Delta Y = \frac{m_m}{k+m_m}(D+E) \qquad (2.1.2)$$

In order to get insight into the tenability of Holtrop's analysis concerning the balance of payments surplus and his conclusion drawn from it, we are investigating now the influence resulting from the monetary impulses on this balance of payments. The balance of payments surplus or deficit will be equal to the difference between the external monetary impulses on one side and the outflow of liquidity to abroad as a result of payments for imports induced by monetary impulses on the other side:

$$B = E - m_m \Delta Y \qquad (2.1.3)$$

If we write the definition of the balance of payments surplus (2.1.3) in terms of domestic and external monetary impulses we will get after some substitution with the aid of (1.2.8) and (2.1.2) the identity:

$$B = \frac{E - m_b D}{1+m_b} \qquad (2.1.4)$$

The condition for the balance of payments equilibrium that can be obtained is that the ratio between external and domestic monetary impulses has to be equal to the value m_b.

$$\frac{E}{D} = m_b \qquad (2.1.5)$$

After examination it will become evident that in (2.1.4) as well as in (2.1.5) the condition of dimensional homogeneity was fulfilled. This is not the case in Holtrop's theory, who erroneously formulates the condition for the balance of payments equilibrium as $E/D = m/k$, which, such as was

demonstrated in the preceding chapter, proves to be the result of using the marginal propensity to import in an incorrect way [16].

Before focussing our attention on the statistical verification it may be noted that instead of the condition of the balance of payments equilibrium (2.1.5) Holtrop could better have formulated the general identity of the balance of payments surplus (2.1.4), because:

1. the balance of payments equilibrium does not always need to be the purpose to which monetary policy should conform. Under the influence of objections with regard to the domestic economic situation a certain balance of payments surplus or deficit can consciously be pursued.

2. the general formulation is verifiable in an empirical way. This is in constrast with the condition for the balance of payments equilibrium because in a strict sense balance of payments equilibrium never occurs.

If for this reason Holtrop had used our general formulation with respect to the balance of payments surplus (2.1.4) then he conceivably could have reached another conclusion.
Finally it should be noted that the marginal propensity to import is liable to such strong fluctuations that a criterion of policy based on the ratio between external and internal monetary impulses, which assumes a prognostical marginal propensity to import, can be viewed as useless as far as all practical application is concerned. In the years between 1954 and 1969 the marginal propensity to import in the Netherlands fluctuated, if the year 1958 (m_b = -1,55) is left out of consideration, between 0,29 in 1963 and 0,80 in 1957.
However it may be expected that a somewhat greater stability of the marginal propensity to import will be attained if the imports are corrected for the changes in the stock-piling.

16) Ibid., p. 287.

In conclusion it may be postulated that the identity in question in the particular shape of the balance of payments equilibrium (2.1.5), as used by Holtrop, even when correctly formulated, has no practical sense. In the general form of the balance of payments surplus or deficit (2.1.4), which we preferred, there remains only the insignificant possibility to define the marginal propensity to import ex-post factor. The latter can be identified much easier from its definition:
the change in the level of imports as a rate of the change in the level of income.

§2. The statistical verification.

Let us now focus our attention on the statistical verification that Holtrop is giving of his model in order to get, among other things, an answer to the next question: is the fact that the sum of the monetary impulses can be seen as the cause of the changes in the level of income and imports statistically verifiable?
For that purpose we start from our definition (2.1.1).

$$\Delta Y = \frac{D+E}{k+m_m}$$

Written in another way and taking into account (1.2.8) we will get the next equation:

$$\frac{k}{M}(\Delta Y + m_b \Delta Y) = \frac{D}{M} + \frac{E}{M}$$

which rewritten yields the following relation:

$$\frac{\Delta Y}{Y} + \frac{\Delta Im}{Y} = \frac{D}{M} + \frac{E}{M} \qquad (2.2.1)$$

in which M represents the domestic stock of liquidity and Y the level of national income.
This relation between the sum of the changes in the level of income and imports as a proportion of the level of income on

the one side and the sum of domestic and external monetary impulses as a proportion of the stock of liquidity on the other side proves to be a definition in which both the regression coefficients of the domestic and external monetary impulses have the value 1.
A statistical verifications of (2.2.1) would therefore be fully senseless because it is a certainty in advance that the two regression coefficients are equal to 1 and will be attented by a correlation coefficient of 100%.
In addition a calculation of this identity has been included in appendix III.
Nothing is less interesting than the statistical verification of an identity. Moreover in this relation the monetary impulses, that are considered as the cause of the changes in the level of income and imports, are not or hardly controlable entities. Holtrop does use this identity but leaves, in his analysis, the activation of liquidity out of consideration and consequently verifies statistically the following relation

$$\frac{\Delta Y}{Y} + \frac{\Delta Im}{Y} = f\left\{\frac{Lcr}{M} + \frac{E}{M}\right\} .$$

In advance it will be clear that on verifying this incomplete identity the coefficients of regression will deviate from its value 1 and that the coefficients of the correlation become less than 100%. The extent in which the activation of liquidity will have played a part is determining for the extent of correlation.
That Holtrop obtains a "high" degree of correlation of 75% in our opinion only implies that not involving the activation of liquidity into the analysis did not affect the results too much.
No economic interpretation may be attached to the regression coefficients of the creation of liquidity and of the external impulses themselves because they deviate from the value 1 under the influence of leaving out of consideration the activation of liquidity.

If we are following now the conclusions which are drawn by Holtrop, which he bases on his regression-analysis, and taking into account that the verification of a nearly correct identity as a matter of course produces an almost complete correlation, then we can make the following comments:[17)]

1. That the high degree of correlation which was found "is the more satisfactory since the factor liquidity activation had to be let out of the consideration" [18)] unfortunately must be rejected as an incorrect formulation.
   If the liquidity activation is included into the regression-analysis a complete correlation will result. Should the liquidity activation be left out of consideration, then the correlation will be violated to the extent in which the activation of liquidity has been of importance.

2. That "the analysis gives no support to the thesis that changes in the quantity of money will be compensated by reverse changes in the velocity of the circulation", [19)] is correct.
   Unfortunately in the results of this analysis we cannot find any support for the contrary. Only the beginning of an indication that the activation of liquidity was possibly left behind somewhat in an absolute sense by the other monetary impulses can be gathered from the results as found by Holtrop,but by no means the direction in which the activation of liquidity could have accompanied the liquidity creation.

3. If we take into consideration the forthgoing then it seems to be doubtful that "the coefficients of regression that where found indicate that a greater influence on the increase of national income and of imports would be exerted

17) See for further figures, Appendix II.

18) Ibid., p. 293.

19) Ibid., p. 293.

by external than by domestic monetary impulses". [19]
From the following comments it will become evident that this has to be considered as an incorrect conclusion. The cause of the fact that on the whole the coefficients of the regression of external impulses will show a smaller deviation of the value 1 than the coefficient of regression of the liquidity creation can be proved mathematically. We found that the cause must be sought in the relation in value between creation of liquidity and external impulses. This can be made plausible as follows: the regression coefficient of that factor that relatively exercises the most important influence will deviate the least from the value 1 resulting from the definition.
In the course of the year 1954-1969, the absolute values of external impulses and the creation of liquidity are roughly speaking in the ratio E: Lcr = 2:1. Consequently, the value of the coefficient of regression of the external impulses E will prove,in general,to remain closer to the value 1 than the regression coefficient of the creation of liquidity Lcr.
Further it can be put forward that it is known from the definition that the values of the sum of changes in the level of income and imports are lying in the plane of explanation shaped by the total domestic and external monetary impulses.
The classification of these values within this plane of explanation, a classification that is not structured, may affect the coefficients of regression and correlation that result from the analysis based on another plane shaped by the external monetary impulses and the domestic liquidity creation. The inequality of the values of the regression coefficients proves to be based therefore entirely on the fact that a (smaller) part (i.e. the activation of liquidity) of one

19) Ibid., p. 293.

(the smallest) of the two monetary impulses (i.e. the domestic monetary impulse) of the definition, when verifying it statistically, is left out of consideration.
For this reason we consider it theoretically fully correct and explicable that the assumption of equal elasticities of national income and imports with regard to internal and external impulses are underlying the monetary model of the Nederlandsche Bank. After the preceding comments, further research on this point, as suggested by Holtrop, now seems to be superfluous.
Finally, we wish to make a comment on Holtrop's last conclusion.[20)]

4. That "the analysis does not give any positive answer to the problem of the direction of the causal relationship between impulses and effects implied in the model of the Bank" can not be considered as a conclusion resulting from the regression analysis because there is no regression analysis that can conclude with respect to the direction of a causal relationship.

In summary it must be emphasized that none of the above conclusions and comments that Holtrop was drawing from the results of his regression analysis are entirely correct or relevant. As long as the model that was used, as we discussed in chapter I, consists merely of truisms based on the "identity approach" every statistical test will yield again a complete correlation. When testing an equation not fully corresponding to the given definitions, the correlation will be frustrated to the extent in which one was deviating from the definition.

Summary.

When recapitulating our observations we find that the differences between the real multiplier and the monetary

20) Ibid., p. 294.

multiplier do not result from, as stated by Kessler, the use of the "identity approach" and the "equality approach" only. If an appropriate multiplier-analysis will be made, the change in an endogeneous variable as a result of the multiplier-process has to be expressed in terms of the change in the exogenous variable that was the cause of the multiplier-process. From our analysis, supplemented by Holtrop's figures, it appears that the "identity-approach" in the monetary analysis, even under stringent assumptions with regard to the effectiveness of monetary policy, can hardly be considered as a multiplier-analysis. It degenerates into an identity from which one can get only ex-post some insight into the marginal cash-quota which in fact is fairly unstable.
An analysis based on the "equality-approach" applied to both multipliers clearly illustrates that they are describing the same process.
When a symmetrical analysis is applied, differences suggested to be essential will fade away.
Essential differences between the real and the monetary multipliers will arise, however, in a dimensional-analysis. For now the dimensional inconsistency of the model such as presented by Holtrop is becoming evident. However, if we consider the marginal propensity to import somewhat closer then this proves to be formulated in such a way that there is no place for it in the monetary part of the analysis. Though it was never distinguished, the monetary marginal import-leakage and the marginal propensity to import prove to be two different entities. The ratios k and m_m playing a part in the monetary analysis prove to be entities which have to be expressed in the dimension of time by which the value of m_m is defined in fact as well as by the length of the period under consideration. This was not the case at all for the ratios s and m_b which play a part in the real multiplier because it was demonstrated that they are dimensionless entities.
In this way the difference was stated between the marginal propensity to import m_b used in the analysis of the spending

process and the monetary marginal import-leakage m_m used in the monetary analysis. Afterall the relation between both of them was also formulated. The relation between the values of the monetary marginal import-leakage and of the marginal propensity to import proved to be equal to the value of the marginal cash-quota k.

As a result of this some identities from the monetary model of the Nederlandsche Bank were rewritten in chapter II and moulded in the same shape as the figures given by the Nederlandsche Bank in order to be able to judge the importance of the statistical verifications.

In the last paragraph we demonstrated the high correlations in Holtrop's statistical verification may not be interpreted in a way other than that the relation that was tested was not frustrated altogether too much by the deviation of the definition underlying it.

Finally we question whether after all the comments we made about the monetary analysis, the analysis based on identities such as used by the Nederlandsche Bank (provided that it is formulated in a correct way) could be an ex-post aid in defining the effectiveness of the measures taken in the framework of monetary policy. At any rate, in our opinion it forms an important source of inspiration for those who are trying to enlarge the monetary instruments to effective policy-tools on the basis of which an active and quantitatively reliable monetary policy can be pursued.

To achieve that purpose it is our opinion that a number of new theoretical insights be pursued in order to formulate a more accountable analysis in a dynamic way.

APPENDIX I.

Years		Domestic liquidity creation of behalf of Government		Other domestic liquidity creation	
		Years		Years	
A*	B*	A*	B*	A*	B*
1954		-2.5		4.2	
1955		-1.5		5.0	
	1956		1.1		3.5
	1957		6.0		0.1
1958		1.5		-3.6	
1959		-7.0		2.6	
1960		-8.1		1.9	
	1961		-3.2		5.7
	1962		3.7		2.9
1963		-0.7		5.2	
	1964		2.3		6.8
	1965		2.7		5.3
	1966		5.7		1.3
1967		2.9		7.4	
1968		4.4		9.3	
	1969		2.4		4.9
Average		-1.4	2.6	4.0	3.8

A* = without credit restriction

B* = with credit restriction

Source: De Nederlandsche Bank N.V., Report for the year 1969, Statistical annex, table 4.3., Amsterdam, April 1970.

Borrowed from: On the effectiveness of monetary policy: the experience of The Netherlands in the years 1954-1969, in Money in an open economy, N.W. Holtrop, Leiden 1972 pp. 253.

Total domestic liquidity creation		Liquidity activation		Total donestic monetary impulses	
Years		Years		Years	
A*	B*	A*	B*	A*	B*
1.7		.6.0		7.7	
3.5		2.7		6.2	
	4.6		11.8		16.4
	5.9		-0.3		5.6
-2.1		-11.4		13.5	
-4.4		4.3		-0.1	
-6.2		6.1		-0.1	
	2.5		0.8		3.3
	6.6		1.1		7.7
4.5		2.9		7.4	
	9.1		7.1		16.2
	8.0		2.6		10.6
	7.0		2.7		9.7
10.3		-2.1		8.2	
13.7		-3.5		10.2	
	7.3		1.0		8.3
2.6	6.4	0.6	3.4	3.3	9.7

APPENDIX II.

For the verification of identity (2.1.4) we use the figures from the 1969 annual report of the Nederlandsche Bank table 4.3 of the statistical appendix pp. 24 and 25.
The marginal propensity to import is calculated as follows:

$$\frac{\frac{m_m \Delta Y}{M}}{\frac{k\Delta Y}{M}} = \frac{m_m}{k} = m_b$$

in which

$$\frac{m_m \Delta Y}{M}$$

is given in line 1b [21]

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21) In fact the variable $\frac{m_b \Delta Y}{M}$ is given in line 1_b. In addition this factor forms part of the external monetary impulses which should merely be rejected because of dimensional reasons. Having no need to enter into these problems here, which were already treated, we use in this and the following appendix line 1_b as the factor $\frac{m_m \Delta Y}{M}$ through which it will be possible at the same time to use the external monetary impulse E according to table 4.3 of the annual report of D.N.B. Only the marginal propensity import thus obtained has still to be multiplied with the factor k.
However, we are pleading that as well line 1_b with respect to the additional imports as item 7 regarding the external monetary impulses are entered henceforth in a corrected way into the appendix of the annual report in order to make the values appropriate to the monetary survey.

$$\frac{B}{M} = \frac{\frac{E}{M} - m_b \frac{E}{M}}{1 + m_b}$$

then we get next table:

	m_b *)	E/M *)	D/M	B/M	$\frac{\frac{E}{M} - m_b \frac{D}{M}}{1 + m_b}$ *)
1954	1,09	20,1	7,7	5,6	5,6
1955	1,52	16,5	6,2	2,8	2,8
1956	1,64	7,1	16,4	-7,5	-7,5
1957	1,94	5,0	5,6	-2,0	-2,0
1958	-3,69	4,1	-13,5	17,0	17,0
1959	0,86	19,6	-0,1	10,6	10,6
1960	1,62	24,5	-0,1	9,4	9,4
1961	1,20	7,9	3,3	1,8	1,8
1962	0,78	8,5	7,7	1,4	1,4
1963	0,69	12,6	7,4	4,4	4,4
1964	1,18	18,7	16,2	-0,2	-0,2
1965	0,89	11,1	10,6	0,9	0,9
1966	1,00	7,9	9,7	-0,9	-0,9
1967	0,76	8,2	8,2	1,1	1,1
1968	1,27	13,2	10,2	0,1	0,1
1969	1,51	20,6	8,3	3,2	3,2

*) See footnote 21.

APPENDIX III.

In order to verify the identity (2.2.1)

$$\frac{\Delta Y}{Y} + \frac{\Delta Im}{Y} = \frac{D}{M} + \frac{E}{M} \quad \text{or} \quad \frac{k}{M}(\Delta Y + m_b \Delta Y) = \frac{D}{M} + \frac{E}{M}$$

we use again the figures from the annual report 1969 of the Nederlandsche Bank table 4.3 of the statistical appendix pp. 24 and 25.

$\frac{k\Delta Y}{M}$ is given by item 4

$\frac{m_m \Delta Y}{M}$ is given by line 1b*

$\frac{D}{M}$ is given by the total of item 6

$\frac{E}{M}$ is given by the total of item 7 *

	$\frac{k\ \Delta Y}{M}$	$\frac{m_m \Delta Y}{M}$ *)	totaal	D/M	E/M*)	Totaal
1954	13,3	14,5	27,8	7,7	20,1	27,8
1955	9,0	13,7	22,7	6,2	16,5	22,7
1956	8,9	14,6	23,5	16,4	7,1	23,5
1957	3,6	7,0	10,6	5,6	5,0	10,6
1958	3,5	-12,9	-9,4	-13,5	4,1	-9,4
1959	10,5	9,0	19,5	-0,1	19,6	19,5
1960	9,3	15,1	24,4	-0,1	24,5	24,4
1961	5,1	6,1	11,2	3,3	7,9	11,2
1962	9,1	7,1	16,2	7,7	8,5	16,2
1963	11,8	8,2	20,0	7,4	12,6	20,0
1964	16,0	18,9	34,9	16,2	18,7	34,9
1965	11,5	10,2	21,7	10,6	11,1	21,7
1966	8,8	8,8	17,6	9,7	7,9	17,6
1967	9,3	7,1	16,4	8,2	8,2	16,4
1968	10,3	13,1	23,4	10,2	-13,2	23,4
1969	11,5	17,4	28,9	8,3	20,6	28,9

*) See footnote 21.

PREVIOUS NUMBERS:

EIT 1	J. Kriens *)	Het verdelen van steekproeven over subpopulaties bij accountantscontroles.
EIT 2	J. P. C. Kleynen *)	Een toepassing van „importance sampling".
EIT 3	S. R. Chowdhury and W. Vandaele *)	A bayesian analysis of heteroscedasticity in regression models.
EIT 4	Prof. drs. J. Kriens	De besliskunde en haar toepassingen.
EIT 5	Prof. dr. C. F. Scheffer *)	Winstkapitalisatie versus dividendkapitalisatie bij het waarderen van aandelen.
EIT 6	S. R. Chowdhury *)	A bayesian approach in multiple regression analysis with inequality constraints.
EIT 7	P. A. Verheyen *)	Investeren en onzekerheid.
EIT 8	R. M. J. Heuts en Walter A. Vandaele *)	Problemen rond niet-lineaire regressie.
EIT 9	S. R. Chowdhury *)	Bayesian analysis in linear regression with different priors.
EIT 10	A. J. van Reeken *)	The effect of truncation in statistical computation.
EIT 11	W. H. Vandaele and S. R. Chowdhury *)	A revised method of scoring.
EIT 12	J. de Blok *)	Reclame-uitgaven in Nederland.
EIT 13	Walter A. Vandaele *)	Mødscø, a computer programm for the revised method of scoring.
EIT 14	J. Plasmans *)	Alternative production models. (Some empirical relevance for postwar Belgian Economy)
EIT 15	D. Neeleman *)	Multiple regression and serially correlated errors.
EIT 16	H. N. Weddepohl	Vector representation of majority voting.
EIT 17		
EIT 18	J. Plasmans *)	The general linear seemingly unrelated regression problem. I. Models and Inference.
EIT 19	J. Plasmans and R. Van Straelen *)	The general linear seemingly unrelated regression problem. II. Feasible statistical estimation and an application.
EIT 20	Pieter H. M. Ruys	A procedure for an economy with collective goods only.
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EIT 26	Th. van de Klundert *)	Labour values and international trade; a reformulation of the theory of A. Emmanuel.
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EIT 28	A. van Schaik *)	A note on the reproduction of fixed capital in two-good techniques.
EIT 29	H. N. Weddepohl *)	Vector representation of majority voting; a revised paper.
EIT 30	H. N. Weddepohl	Duality and Equilibrium.
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EIT 37	J. Plasmans	Adjustment cost models for the demand of investment.
EIT 38	H. N. Weddepohl	Dual sets and dual correspondences and their application to equilibrium theory.
EIT 39	J. J. A. Moors	On the absolute moments of a normally distributed random variable.

EIT 1973

*) not available